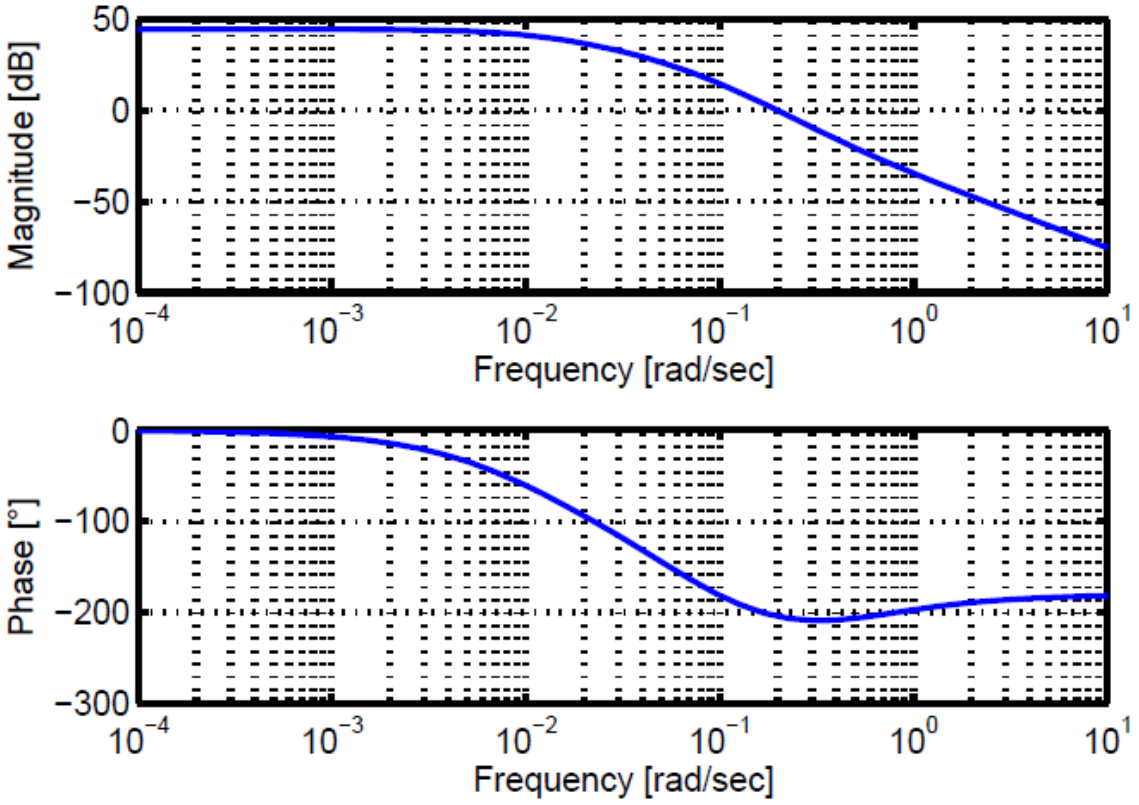
EE 310 - LAB 9

Question:

Assume the transfer function $G(s) = \dfrac{340\,s + 170}{20\,000\,s^3 + 3\,200\,s^2 + 130\,s + 1}$ is given.

a. Determine the pole-zero representation and the time-constant representation of $G(s)$.

b. The following figure shows the bode plot of $G(s)$. Assume that the input signal $u(t) = 5\sin(0.02\,t) + 3\sin(0.1\,t + \pi/2)$ is applied to the transfer block with transfer function $G(s)$. Determine the output signal $y(t)$ using the bode plot.



c. Verify the magnitude and phase values in the bode plot for the frequencies $\omega = 0.02, 0.1, 0.2, 1$.

Solution:

a. There is one zero at $s = -0.5$ and there are poles at $s = -0.1$, $s = -0.05$ and $s = -0.01$. The pole-zero representation and time-constant representation are

$$G(s) = 0.017\,\frac{s + 0.5}{(s + 0.1)(s + 0.05)(s + 0.01)} \qquad G(s) = 170\,\frac{1 + 2\,s)}{(1 + 10\,s)(1 + 20\,s)(1 + 100\,s)}.$$

b. The interesting frequencies are 2 rad/sec and 10 rad/sec. For 0.02 rad/sec, the magnitude is $|G(j\,2)|_{dB} \approx 40\,\text{dB} \hat{=} 100$ and the phase is $\angle(G(j\,2)) = -90° \hat{=} -\pi/2$ rad. For 0.1 rad/sec, the magnitude is $|G(j\,10)|_{dB} = 20\,\text{dB} \hat{=} 10$ and the phase is $\angle(G(j\,10)) = -180° \hat{=} -\pi$ rad. Hence, we get the output signal

$$5 \cdot 100\sin(0.02\,t - \pi/2) + 3 \cdot 10\,\sin(0.1\,t + \pi/2 - \pi).$$

Hint:

- Consider output computation in the Laplace domain:
  $Y(s) = G(s)\,U(s)$
- Sinusoidal input function: $U(s) = \dfrac{\omega}{s^2 + \omega^2}$

Frequency Response: Result

Result

$$y(t) = |G(j\omega)| \sin\left(\omega t + \angle(G(j\omega))\right)$$

Description

- Output signal y oscillates with same frequency ω as input signal u
- Amplification of u by $|G(j\omega)|$
- Phase shift of u by $\angle(G(j\omega))$

Bode Plot: Basic Idea

Description

- Given: Transfer function $G(s)$
- Task: Show the frequency response in terms of magnitude $|G(j\omega)|$ and phase shift $\angle(G(j\omega))$

Magnitude Plot

- Frequency axis with logarithmic scale ω [rad/sec]
- Magnitude axis with $20 \log |G(j\omega)|$ [dB]

Phase Plot

- Frequency axis with logarithmic scale ω [rad/sec]
- Phase axis with $\angle G(j\omega) = \arctan(\dfrac{\mathrm{Im}(G(j\omega))}{\mathrm{Re}(G(j\omega))})$ [°]