EE 310 - LAB 8

Question:

We consider the basic feedback loop with the open-loop transfer function $G_o(s) = K \frac{1}{s\,(s^2 + 2\,s + 5)}$.

a. Sketch the root locus of for $G_o(s)$.

Hint: Also find the intersection of the root locus with the imaginary axis

Simulate a reference step response of the feedback loop

Solution:

The following rules were used:

R1 The root locus has $3 - 0 = 3$ branches

R2 The root locus starts at $s = 0$, $s = -1 \pm 2j$ and ends at $|s| \to \infty$

R3 The root locus stays on the real axis at the left of $s = 0$

R4 The root locus has 3 asymptotes:

$$\sigma = \frac{0 - 1 + 2j - 1 - 2j}{3} = -\frac{2}{3}, \qquad \theta = \frac{\pi}{3}(2\,k + 1)$$

R5 There are no break-away points

R6 Angle of departure for $p_j = -1 + 2j$

$$\pi - \angle(-1 + 2j - 0) - \angle(-1 + 2j + 1 + 2j) = \pi - \angle(-1 + 2j) - \angle(4j) =$$
$$= \pi - 0.65\,\pi - \pi/2 \approx -0.15\,\pi\ (-27^\circ)$$

R7 There are no conjugated complex zeros

R8 The characteristic polynomial of the closed loop is $D(s) + K\,N(s) = s^3 + 2\,s^2 + 5\,s + K$. We compute

$$(s^3 + 2\,s^2 + 5\,s + K) : (s^2 + \omega^2) = s + 2 \text{ remainder: } (5 - \omega^2)\,s + K - 2\omega^2$$

Hence, we conclude $\omega = \sqrt{5}$ and $K = 2 \cdot 5 = 10$. The root locus intersects the imaginary axis at $s = \pm j\sqrt{5}$.

The root locus plot is shown in the following figure.