EE 310 - LAB 7

Question:

Consider the following transfer function:

$$G(s)=\frac{s+4}{(s+7)(s^2+3s+3)}$$

Assume that somebody designed a controller *C(s)* with the following transfer function:

$$C(s)=K\frac{s+7}{(s+1)s}$$

a. Assume that *K=1*. Show that the basic feedback loop with *G(s)* and *C(s)* as given above is internally stable.
b. The complementary sensitivity is as follows:

$$T(s)=\frac{G(s)C(s)}{1+G(s)C(s)}$$

T(s) has the following dominant pair of poles for *K=1*;

$$s_{1,2}=-0.16\pm 0.84j$$

Determine the estimated rise time, peak time and settling time (%2) for the step response of *T(s)*.

c. Simulate *1+C(s)G(s)* in Matlab/Simulink.

<u>Solution:</u>

a. We compute:

$$1+C(s)G(s)=1+\frac{s+7}{(s+1)s}\frac{s+4}{(s+7)(s^2+3s+3)}=1+\frac{s+4}{(s+1)s(s^2+3s+3)}$$
$$=\frac{(s^2+s)(s^2+3s+3)+s+4}{(s+1)s(s^2+3s+3)}=\frac{s^4+4s^3+6s^2+4s+4}{(s+1)s(s^2+s+1)}$$

For stability, we want that all zeros of *1+C(s)G(s)* lie in the OLHP. We can use the Routh-Hurwitz criterion to check this:

S^4	1	6	4
S^3	4	4	
S^2	5	4	
S^1	4/5		
S^0	4		

There is no change of sign in the first column, which means that all zeros of *1+C(s)G(s)* lie in the OLHP. Hence, the feedback loop is internally stable.

b. We compute the polynomial ;

$$(s+0.16+0.84j)(s+0.16-0.84j)=s^2+0.32s+0.7312$$

We get the following values:

$$\omega_n^2 = 0.7317$$
$$\omega_n = 0.8551$$
$$2D\omega_n = 2D0.8551 = 0.32$$
$$D = 0.1871$$

$$t_r = \frac{1}{\omega_n\sqrt{1-D^2}}\arctan(\frac{\sqrt{1-D^2}}{D}) = 1.64$$
$$t_p = \frac{\pi}{\omega_n\sqrt{1-D^2}} = 3.74$$
$$t_s = \frac{4}{\omega_n D} = 25$$

We expect that the output response reaches the final value fast but it takes long until it stays close to the final value.