

EE 310 - LAB 6

Question 1:

Consider the following transfer functions:

$$G_1(s) = \frac{(s + 3.5)(s + 4)}{(s + 3)(s + 10)}$$

$$G_2(s) = \frac{s + 8}{(s - 3)(s + 200)}$$

$$G_3(s) = \frac{s + 2}{(s + 10)(s + 20)}$$

- 1) Determine the dominant pole for each of the transfer functions.

G_1 : The transfer function is stable, hence we look for a stable dominant pole. Although $s = -3$ is the pole with the smallest absolute value, it is suppressed because of the zero at $s = -3.5$ that is very close to the pole. Hence, the pole at $s = -10$ is dominant.

G_2 : The transfer function is unstable, so the unstable pole at $s = 3$ is dominant.

G_3 : The transfer function is stable. The dominant pole is the pole with the smallest absolute value which is at $s = -10$.

- 2) Which of the transfer functions leads to overshoot in the step response?

G_1 has a zero at $s = -3.5$ which is closer to the imaginary axis than the dominant pole at $s = -10$. Hence, we expect overshoot.

- 3) Simulate the step responses of G_1 , G_2 , G_3 and G_4 in Matlab/Simulink.

Hint:

Conditions for Poles:

- If there is an unstable pole, it dominates all stable poles
- Usually, stable poles close to the imaginary axis (slow convergence) dominate stable poles far from the imaginary axis (fast convergence)
- If $z_j \approx p_i$, dominant modes can be suppressed and other modes become dominant

Minimum-Phase Zero:

- Zeros in the open left half plane: $\text{Re}(z_j) < 0$

Non-minimum Phase Zero:

- Zero in the right half plane: $\text{Re}(z_j) > 0$

Overshoot:

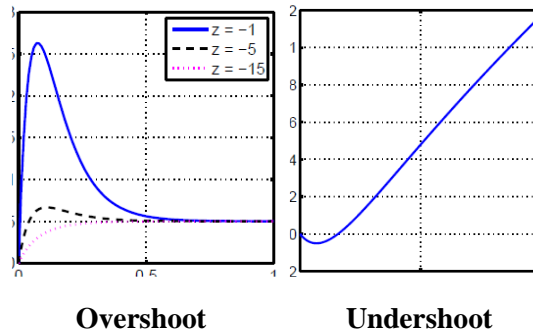
- Dominant plant pole at p with $\text{Re}(p) < 0$
- Slow minimum phase plant zero z with $\text{Re}(p) \ll \text{Re}(z) < 0$

⇒ Overshoot of the step response

Undershoot:

- k non-minimum phase zeros in $G(s)$
- Step response intersects with time-axis k times

⇒ Undershoot whenever there are non-minimum phase zeros



Question 2:

Consider the following transfer function of the modified vehicle suspension system with friction:

$$G(s) = \frac{K A_H}{ms^2 + \gamma s + c}$$

where, $\gamma = 20000N \text{ sec} / m$, $m = 1000kg$, $c = 10000N / cm$, $g = 10N / kg$, $A_H = 15cm^2$, $K = 100N / cm^2 / V$.

- 1) What is the dominant pole of G(s)?

We get the transfer function:

$$G(s) = \frac{1500}{1000s^2 + 20000s + 10000}$$

The poles of G(s) are at $s_1 = -19.5$ and $s_2 = -0.5$. Hence, the pole at s_2 is dominant because it is closer to the imaginary axis.

- 2) Simulate the step response of G_1 in Matlab/Simulink.